THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH 2040 (Second term, 2015-16)
Linear Algebra II
Study Guide for Final Exam

Introduction

This short note is a summary of what we have learned in this course. You are expected to know the topics listed below (all of which are covered in Ch. 1-4, 5.1, 5.2, 5.4, 6.1-6.6, 6.8, 7.1-7.2 of the textbook). A good way to prepare for the final exam is to study the lecture notes and the textbook, and do the exercises in the Practice Problem Sets and past exam papers. All vector spaces are assumed to be finite dimensional over the field $\mathbb{F} = \mathbb{R}$ or $\mathbb{C}$.

1 Computational aspects

- Solve system of linear equations, elementary row/column operations, find the range and null space of a matrix
- Basic matrix operation, for example, compute A^{-1}, $\det A$ ... etc
- Given a matrix or a linear operator, compute its characteristic polynomial, its eigenvalues and eigenvectors.
- Determine if a matrix is diagonalizable by computing the characteristic polynomial and the eigenspaces. If diagonalizable, find an eigenbasis.
- Compute A^k by diagonalization
- Change a linearly independent subset into an orthonormal subset by Gram-Schmidt Orthogonalization Process and normalization
- Find the adjoint of an operator
- Find orthonormal eigenbasis for normal/self-adjoint operators
- Diagonalization of a symmetric bilinear form
- Change a 3×3 or 4×4 matrix A into its Jordan canonical form by change of basis

2 Theoretical aspects

- Background knowledge on vector spaces, subspaces, span and linear independence, basis and dimension, sum and direct sum of subspaces
- Linear transformations and their matrix representations, change of coordinate matrices, rank-nullity theorem, determinants of matrices and their basic properties
- Definition of eigenvalues, eigenvectors and eigenspaces with their basic properties and geometric meanings

- Relationship between diagonalizability, the existence of eigenbasis, multiplicities of eigenvalues and dimension of eigenspaces
- Invariant subspaces and Cayley-Hamilton theorem
- Properties of inner products and norms (Pythagoras theorem, Cauchy-Schwarz inequality and Triangle inequality) and examples of inner product spaces
- Orthogonality, orthogonal projection and orthogonal complement
- Definition and properties of adjoint operators
- Schur's theorem
- Properties of normal/self-adjoint and unitary/orthogonal operators
- Orthogonal operators on $\mathbb{R}^2$
- Spectral theorems and their applications
- Bilinear forms, symmetric bilinear forms and their matrix representations
- Definition and properties of similar, congruent, orthogonally/unitarily equivalent matrices
- Definition of generalized eigenvectors and eigenspaces